General Certificate of Education
January 2009
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

Tuesday 13 January 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

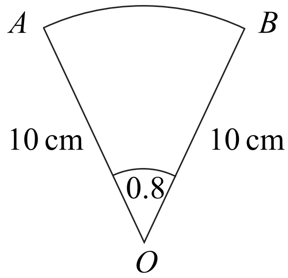
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The diagram shows a sector OAB of a circle with centre O and radius 10 cm.



The angle AOB is 0.8 radians.

(a) Find the area of the sector. *(2 marks)*

(b) (i) Find the perimeter of the sector OAB. *(3 marks)*

(ii) The perimeter of the sector OAB is equal to the perimeter of a square. Find the area of the square. *(2 marks)*

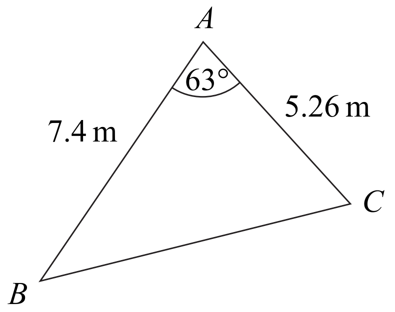
2 (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_{1.5}^{6} x^2\sqrt{x^2-1}\,\mathrm{d}x$$

giving your answer to three significant figures. *(4 marks)*

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

3 The diagram shows a triangle ABC.



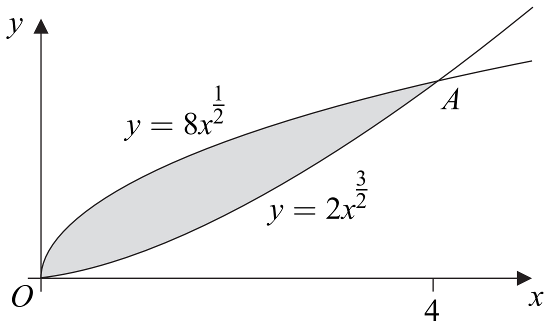
The size of angle A is $63°$, and the lengths of AB and AC are 7.4 m and 5.26 m respectively.

(a) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

(b) Show that the length of BC is 6.86 m, correct to three significant figures. *(3 marks)*

(c) Find the value of **sin *B*** to two significant figures. *(2 marks)*

4 The diagram shows a sketch of the curves with equations $y = 2x^{\frac{3}{2}}$ and $y = 8x^{\frac{1}{2}}$.



The curves intersect at the origin and at the point A, where $x = 4$.

(a) (i) For the curve $y = 2x^{\frac{3}{2}}$, find the value of $\dfrac{dy}{dx}$ when $x = 4$. *(2 marks)*

(ii) Find an equation of the normal to the curve $y = 2x^{\frac{3}{2}}$ at the point A. *(4 marks)*

(b) (i) Find $\displaystyle\int 8x^{\frac{1}{2}}\,dx$. *(2 marks)*

(ii) Find the area of the shaded region bounded by the two curves. *(4 marks)*

(c) Describe a single geometrical transformation that maps the graph of $y = 2x^{\frac{3}{2}}$ onto the graph of $y = 2(x+3)^{\frac{3}{2}}$. *(2 marks)*

Turn over ▶

5 (a) By using the binomial expansion, or otherwise, express $(1+2x)^4$ in the form

$$1 + ax + bx^2 + cx^3 + 16x^4$$

where a, b and c are integers. *(4 marks)*

(b) Hence show that $(1+2x)^4 + (1-2x)^4 = 2 + 48x^2 + 32x^4$. *(3 marks)*

(c) Hence show that the curve with equation

$$y = (1+2x)^4 + (1-2x)^4$$

has just one stationary point and state its coordinates. *(4 marks)*

6 (a) Write each of the following in the form $\log_a k$, where k is an integer:

(i) $\log_a 4 + \log_a 10$; *(1 mark)*

(ii) $\log_a 16 - \log_a 2$; *(1 mark)*

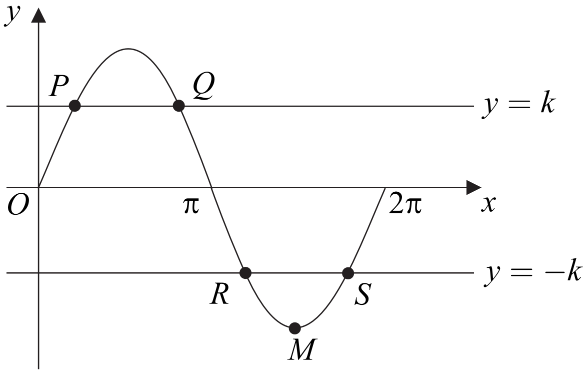
(iii) $3\log_a 5$. *(1 mark)*

(b) Use logarithms to solve the equation $(1.5)^{3x} = 7.5$, giving your value of x to three decimal places. *(3 marks)*

(c) Given that $\log_2 p = m$ and $\log_8 q = n$, express pq in the form 2^y, where y is an expression in m and n. *(3 marks)*

7 (a) Solve the equation $\sin x = 0.8$ in the interval $0 \leqslant x \leqslant 2\pi$, giving your answers in radians to three significant figures. *(3 marks)*

(b) The diagram shows the graph of the curve $y = \sin x$, $0 \leqslant x \leqslant 2\pi$ and the lines $y = k$ and $y = -k$.



The line $y = k$ intersects the curve at the points P and Q, and the line $y = -k$ intersects the curve at the points R and S.

The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

(ii) The x-coordinate of P is α.

Write down the x-coordinate of the point Q in terms of π and α. *(1 mark)*

(iii) Find the length of RS in terms of π and α, giving your answer in its simplest form. *(2 marks)*

(c) Sketch the graph of $y = \sin 2x$ for $0 \leqslant x \leqslant 2\pi$, indicating the coordinates of points where the graph intersects the x-axis and the coordinates of any maximum points. *(5 marks)*

8 The 25th term of an arithmetic series is 38.

The sum of the first 40 terms of the series is 1250.

(a) Show that the common difference of this series is 1.5. *(6 marks)*

(b) Find the number of terms in the series which are less than 100. *(3 marks)*

END OF QUESTIONS

There are no questions printed on this page

There are no questions printed on this page

There are no questions printed on this page